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The Number of Regimes Across Asset Returns:

Identification and Economic Value

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The Number of Regimes Across Asset Returns: Identification and Economic Value

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Abstract

A shared belief in the financial industry is that markets are driven by two types of regimes. Bull markets would be characterized by high returns and low volatility whereas bear markets would display low returns coupled with high volatility. Modeling the dynamics of different asset classes (stocks, bonds, commodities and currencies) with a Markov-Switching model and using a density-based test, we reject the hypothesis that two regimes are enough to capture asset returns' evolutions for many of the investigated assets. Once the accuracy of our test methodology has been assessed through Monte Carlo experiments, our empirical results point out that between two and five regimes are required to capture the features of each asset's distribution. Moreover, we show that only a part of the underlying number of regimes is explained by the distributional characteristics of the returns such as kurtosis. A thorough out-of-sample analysis provides additional evidence that there are more than just bulls and bears in financial markets. Finally, we highlight that taking into account the real number of regimes allows both improved portfolio returns and density forecasts.

JEL Classification: G11, G15, G22
Keywords: Bull and bear markets, Markov switching models, Number of regimes, Density based tests

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1 Introduction

Financial asset prices fluctuate in a tick-by-tick fashion in response to a globalized news flow. The nature and frequency of this flow has a tremendous impact on the dynamics of financial markets, as reflected by the evolutions of the major indices used by financial data providers and newspapers to summarize the flavor of the financial week. In this summarizing process, two kinds of episodes are usually identified and used as labels: a growing valuation of risky assets in a low volatility environment is referred to as a “bull market”. On the contrary, “bear market” is the wording used to describe a period during which government bonds are used as safe havens whereas risky assets deliver strongly negative returns and volatility experiences a sharp increase.

This remains, however, a rough simplification of a much more complex reality and the subject suffers from a lack of attention despite its obvious interest for both academics and practitioners. For example, it is essential to many investment managers to identify the dawn of a new bull trend, especially after a strongly negative bear market such as that experienced during the fourth quarter of 2008 (see for example the Gordy (2000)’s credit risk analysis, Christensen et al. (2004) for bank capital requirements, Bruche and Gonzlez-Aguado (2010) or Ielpo (2011) in the case of the credit cycle). In the meantime, not every financial asset follows strong and persistent bull and bear phases: we have limited empirical evidence of the kind of dynamics that drive for example various currencies. The same is true when it comes to commodities. This article tries to gather cross asset evidence around these market cycles.

A large stream of literature based on Hamilton (1989)’s Markov switching model to investigate the business cycle usually assumes ex ante the number of regimes agitating the economy. Examples include Goodwin (1993), Ghysels (1994) or Kim and Nelson (1999). More recently, researchers turned to this modelling approach to document the behavior of financial markets. Ang and Timmermann (2011) provide a survey of the literature. Ang and Beckaert (2002) present an asset allocation strategy based on a MS(2) model, underpinning the economic performance of such a model when compared to a single regime one. Maheu and McCurdy (2000) present a variaton of a MS(2) model that provides evidence about the duration of each market cycle. Chauvet and Potter (2000) use this modelling approach to build coincident and leading indicators. However, most of these articles focus on equity markets. There is a clear need for empirical works describing and characterizing financial market cycles from a cross asset perspective.

Beyond the insight regarding the time series dynamics of returns, this approach suffers from one drawback: the number of market regimes is assumed before the estimation – even though we have a limited amount of information with respect to this. As in the business cycle case, many articles assume that two regimes are enough to correctly capture the evolution of the main equity indices. See for example Al-Anaswah and Wilfling (2009), Henry (2009), Al-Anaswah and Wilfling (2011) or Dionne et al. (2011). More recently, Maheu and McCurdy (2011) describe how extanding the assumed number of regimes is essential to our understanding of financial markets: using a large dataset of weekly returns, they estimate a four regime model, that involves a bull, a bear, a bearish bull and a bullish bear regimes. Their model provides a better approximation to the daily reality of portfolio managers, as indicated by the various tests considered in their article.

What is more, given the complexity and the variety of the information reflected in asset prices, the economic intuition is supportive to the idea that there are more than just bulls and bears in financial markets. This article intends to fill a gap: we present and compare various tests to determine the number of regimes implicit in returns based on a conditional density argument when the underlying model is a standard Markov Switching model with n regimes.

A Monte Carlo test shows that a test based on the empirical likelihood surface leads to a better estimation of the number of regimes. When turning our attention to a large dataset of weekly returns on several indices, we show that only two foreign exchange rates - namely the Swiss Franc and the Yen versus Dollar - can be modeled by a two-regime MS model. For most of the assets covered here, the number of regimes is higher than 2. The additional regimes are of various natures and asset dependent. For selected cases, the number of regimes can be equal up to 5, as it is the case for the US and European High Yield index. We discuss the persistence and performances under each regime, underlining that the bull/bear specification may be oversimplifying a complex reality. After that, we show that the number of states estimated has only a weak link with the distributional properties of returns. Finally, we provide two different out-of-sample analyses regarding the importance of an accurate measure of the right number of regimes. First, we show that estimating the number of regimes leads to a better prediction of expected returns for many assets. Then, the density forecasts obtained with the estimated number of regimes are equivalent or better than the ones obtained with a two-regime model.

This article is structured as follows: Section 2 presents the underlying modeling methodology, jointly with the Monte Carlo evaluation of the tests used to estimate the number of regimes. Section 3 discusses the elements involved in our dataset. Section 4 presents a detailed discussion of the results. Section 5 assesses the economic value of a higher number of regimes. Section 6 concludes.

2 Testing for the number of regimes implied by financial return dynamics

This section is devoted to the presentation of the material needed for the tests used in this article. First, we briefly review the basics of Hamilton (1989)'s switching model before turning to the presentation of several specification tests.

2.1 A brief presentation of the Markov Switching model

We provide the reader with a short presentation of Hamilton (1989)'s markov switching model. This model was initially introduced in the literature by focusing on the US business cycle. Its use to estimate the regimes driving financial markets has been since developped in various articles such as Chauvet and Potter (2000), Ang and Bekaert (2002a,2002b) or more recently Maheu et al. (2011). This time series model aims at modeling and estimating the changes in regimes that affect different kinds of economic series. It relies on the assumption that the probability to move from one regime to another varies over time, while the transition probabilities are constant.

We present the basic intuitions using a two-regime MS model before turning to a more general case. Let r_t be the logarithmic return on a given asset at time t, for the holding period between $t-1$ and t. Let s_t be an integer value variable that is equal to 1 (respectively 2) at time t if regime 1 (respectively 2) prevails in the economy. Given that the regime i prevails, the conditional distribution of returns is as follows:

$$r_t \sim N(\mu_i, \sigma_i). \tag{1}$$

The probability to be in regime 1 at time t can be written as:

$$P(s_t = 1) = P(s_t = 1|s_{t-1} = 1) \times P(s_{t-1} = 1) + P(s_t = 1|s_{t-1} = 2) \times P(s_{t-1} = 2). \tag{2}$$

$P(s_t = 1|s_{t-1} = 1)$ is assumed to be constant and equal to p, and $P(s_t = 2|s_{t-1} = 1) = 1 - p$. With a similar argument, $P(s_t = 2|s_{t-1} = 2) = q$ and $P(s_t = 1|s_{t-1} = 2) = 1 - q$. These transition probabilities can be gathered into a transition matrix as follows:

$$\Pi = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}, \tag{3}$$

such that

$$\mathbf{P}_t = \Pi \mathbf{P}_{t-1}, \tag{4}$$

with $\mathbf{P}_t = (P(s_t = 1), P(s_t = 2))^{>}$. The parameters driving the model are thus the moments associated with asset returns for each state and the matrix Π. The usual estimation strategy is a maximum likelihood one, based on the filtering approach developed in Hamilton (1989). This two-regime case can be generalised to a $n-$ regime one: in such case, s_t can take integer values ranging from 1 to n, and the Π matrix becomes a $n \times n$ matrix.

2.2 Testing for the number of regimes in a MS model

The literature provides us with various tests to estimate the number of relevant regimes driving time series. Three types of tests can be found in the literature : (1) penalized likelihood tests, (2) Kullback-Leibler distance based tests and (3) tests based on the empirical likelihood surface.

Psaradakis and Spagnolo (2003) consider methods based on complexity-penalized likelihood criteria. They highlight that an estimate of the number of regimes $\hat{n}$ is obtained through:

$$\hat{n} = \arg\max_{1 \leq k \leq k?} \left\{ \log \mathbf{f}_{n_k}\left(r_t; \widehat{\theta}_{n_k}\right) - C_T \dim(\Theta_k) \right\}, \tag{5}$$

where $\log \mathbf{f}_{n_k}\left(r_t; \widehat{\theta}_{n_k}\right)$ stands for the log-likelihood function and Θ_k is the parameter vector for a number of regimes n_k. $C_T \dim(\Theta_k)$ corresponds to the penalization factor. Akaike (1974, 1976), Schwarz (1978) and Hannan-Quinn (1979) choose different values of C_T for selecting the most parsimonious correct model:

– If $C_T = 1$, the Psaradakis and Spagnolo (2003) approach is closed to the AIC;

– If $C_T = \frac{1}{2}\ln T$ the criterion corresponds to the BIC;

– If $C_T = c \times \ln\ln(T)$, $c > 1$, the approach is similar to the Hannan-Quinn criterion. Practically speaking, Psaradakis and Spagnolo (2003) set $c = 1$.

Close to the penalized likelihood criterion, the Kullback-Leibler divergence is another criterion proposed in the literature. Smith et al. (2005) use the following criterion:

$$MSC = -2\log f\left(X, \hat{\theta}\right) + \sum_{t=1} \frac{\hat{T}_t\left(\hat{T}_t + \lambda_t \times K\right)}{\delta_t \times \hat{T}_t - \lambda_t \times K 2}, \tag{6}$$

where $\log f\left(X, \hat{\theta}\right)$ is the maximized log-likelihood value, $\hat{T}_t = \mathbf{tr}\left(\hat{W}_t\right)$, $\hat{W}_t = \text{diag}\,(\zeta_{1t}, \ldots, \zeta_{1T})$, $\delta_t = E\,[\pi_t^?/\hat{\pi}_t]$, $\lambda_t = E\left[(\pi_t^?/\hat{\pi}_t)^2\right]$, and $\pi_t^?$ is the i-th element of the principal eigenvector of $P^?\pi^? = \pi^?$ for the best estimates $\theta^? = \arg\min_\theta E\left[-\log f\left(X^?, \hat{\theta}\right)\right]$.

Another approach to test the number of regimes is based on the empirical likelihood surface as presented by Hansen (1992). Nevertheless, because of important technical issues (Cho and White (2007)) and of computational burden (Psaradakis and Spagnolo (2003)) of this approach – especially when it comes to Markov switching models with three regimes or more – we discarded its implementation.

Here, we add a fourth approach, based on a goodness of fit test. It is based on Vuong (1989)'s test and is related to Diebold and Mariano (1995)'s and Amisano and Giacomini (2007)'s work. It remains connected to Hansen (1992) and Cho and White (2007) in so far as is makes use of the empirical likelihood surface, while being computationnally less burdensome.

Let $f_{n_1}(r_t; \hat{\theta}_{n_1})$ be the likelihood function associated with an estimated Markov-Switching model with n_1 states. Let $f_{n_2}(r_t; \hat{\theta}_{n_2})$ be a similar quantity in the case of a MS model with n_2 regimes. θ_{n_i} is the vector of the parameters to be estimated by maximum likelihood in the n_i-regime case. The two specifications are compared through their associated log density computed with the estimated sample. Let $z_t^{n_1,n_2}$ be the following quantity:

$$z_t^{n_1,n_2} = \log f_{n_1}(r_t; \hat{\theta}_{n_1}) - \log f_{n_2}(r_t; \hat{\theta}_{n_2}).$$

The approach proposed here is based on the following test statistics:

$$t_{n_1,n_2} = \frac{\frac{1}{T}\sum_{t=1}^{T} z_t^{n_1,n_2}}{\hat{\sigma}_{n_1,n_2} \sqrt{T}}, \tag{7}$$

where T is the total number of available observations in the sample used to estimate the parameters, and $\hat{\sigma}_{n_1,n_2}$ a properly selected estimator of the standard deviation of $\frac{1}{T}\sum_{t=1}^{T} z_t^{n_1,n_2}$. Several methods have been proposed in the literature to estimate the standard deviation. We propose to use a Newey West (1987) estimator by considering different lags.

Under the null hypothesis that both models provide an equivalent fit of the returns' distribution, Amisano and Giacomini (2007)'s Theorem 1 provides the asymptotic distribution of this test statistic:

$$t_{n_1,n_2} \sim N(0, 1). \tag{8}$$

This test statistic allows us to compare non nested and nested models[1]. We assume that the conditions stated in Theorem 1 of Amisano and Giacomini are granted in our case.

The main issue with this kind of test is that when comparing in-sample fits of the distribution, the bigger the number of parameters and the better the fit of the distribution obtained. Hence, by basing our analysis on an in-sample test of MS models with a number of regimes ranging between 1 and 10, we are very likely to decide that the model with 10 regimes should be retained for it provides us with the best fit possible. To circumvent this problem, we propose retaining the number of regimes that delivers the best fit from the previous statistics point of view, while being as parsimonious as possible. We retain the optimal number of regimes, $\hat{n}$, such that : $t_{\hat{n},\hat{n}_{n-1}} \geq q_\alpha$ and $t_{\hat{n}_{n+1},\hat{n}_n} < q_\alpha$, with q_α being the quantile for a given risk level of α%. By doing this, we increase the number of regimes as long as it statistically improves the fit of the returns' distribution. On the contrary, among of a set of statistically equivalent models we retain the specification with the lowest number of regimes, to avoid the "overfitting" issue.

2.3 A Monte Carlo investigation of the test methodologies

Before applying the previous methodologies to a real dataset of financial returns, we perform a Monte Carlo assessment of the previous tests. We want to gauge the ability of the tests to estimate the number of underlying regimes when this number is known and the model is a Markov Switching model. Psaradakis and Spagnolo (2003) provide a similar exercise on a 1- vs 2-regimes case. We extend this empirical assessment to a MS model with a higher number of regimes, as additional estimation issues are likely to arise in such a situation. We use the specifications presented in Maheu and Mc Curdy (2011) who propose a latent 4− state Markov-switching model for weekly stock returns. This model presents a special interest for our work, as it differs from the usual bull-bear dyptic: the test would behave very badly if it was to diagnose two underlying regimes when there are indeed more.

The detailed characteristics of the model are provided by Tables (1) and (2). They illustrate the component states of bull and bear market regimes. The bear and bear rally states – regimes 1 and 2 – govern the bear regime; the bull correction and bull states – regimes 3 and 4 – govern the bull regime. One of the main features of a latent 4-state model is the possibility to consider short-term reversals within each regime of the market. For example, the bear regime is characterized by strong negative returns – regime 1's average return is equal to -0.92% – but can exhibit persistent rallies – regime 2's average return is equal to 0.23%. Similarly, even if the long-run return of the bull regime is positive, it can be characterized by negative returns during bull corrections. We also note that the bear regime has the highest volatility while the bull one has the lowest. The transition probabilities reflect the strong persistence of each state: for example, state 2 has the highest persistence (p_{22} = 96.80%). Nevertheless, this state can be transitory: from state 2, the market goes either to state 1, a bear state, with a probability of 1.4%, or to state 4, a bull state, with a probability of 1.8%. A similar interpretation can be reached for bull corrections: in this case, the market enters a bear regime with a probability equal to 0.9%, or a bull regime, with a probability equal to 4.8%. The transition probabilities display also some asymmetries. For example,

[1] This is true as long as we independantly reestimate the competing models.

the probability that the market switches from a bull phase to a state characterized by low returns and high volatilities is close to zero ($p_{32} = p_{41} = 0\%$ and $p_{42} = 0.3\%$) whereas the probability of switching from a bear phase to a bull market is close to 7% ($p_{14} = 6.91\%$).

According to this specification, we simulate 1000 samples of 650 weeks of returns. That is 12.5 years of returns, which involves, in our view, enough market episodes to provide us with a rather structural view about the regimes affecting asset returns. What is more, this sample size is consistent with the real dataset that we intend to use. For each of the simulated samples, we reestimate the MS(i) parameters, i ranging from 1 to 10. Once the parameters are estimated, we run each test to select the optimal number of regimes.

Table (3) provides the results of the different test methodologies. The penalized likelihood-based criteria tends to underestimated the right number of regimes: for more than 90% of the cases the number of regimes is underestimated whereas the results for the Kullback-Leibler divergence are similarly under- or overestimated (about 45% of the cases). The likelihood ratio test at 1% threshold, with a Newey-West variance estimator computed with 0 lag, appears to be the best test available, with a success ratio equal to 80%. As highlighted in Psaradakis and Spagnolo (2003), the complexity-penalized likelihood methods fail to correctly estimate the number of regimes. The tests select the right number of regimes in only 10% of the cases. Regarding the likelihood ratio test, the accuracy obviously depends on the risk level at which the test is performed. The lower the risk level, the higher the percentage of correcly estimated number of regimes. This could be related to the fact that with a lower risk level the test has a tendancy to be more conservative and to impose a stronger penalization on a higher order Markov switching model. With a 10% risk level, the number of cases for which the number of regimes is overestimated is multiplied by 3 when compared to a 1 to 5% case. For example, with the likelihood ratio test at a 10% threshold, 30% of the number of regimes are overestimated and 10% are underestimated whereas at a 5% level, the results are respectively 12% and 15%.

Given the previous results, we focus on the likelihood ratio tests performed with the highest confidence level, that is 1% in the empirical applications presented hereafter.

3 Dataset

The dataset used here jointly with the statistical test presented earlier is made of a wide scope of financial assets. Every data has been obtained from Bloomberg.The dataset encompasses four types of assets:

– Equities: we consider different types of equity indices covering different regions of the world. First, we consider both large and small capitalization indices for the developped world stock market. In the US case, the large caps index is chosen to be the SP500 and the small caps one is the Russel 3000. In the EMU case we use Eurostoxx 50 and the MSCI small caps EMU. In the emerging case, we focus our attention on broad indices that are segregated depending on geographic arguments: we use the MSCI EM Asia, Latam and Europe.

– Bonds: we cover three types of bond indices in the developped market case. The first one is Government bond indices both in the US and in the EMU case. The second and third ones are investment grade and high yield bond indices for both the US and the EMU. These indices are Bank of America Merrill Lynch's indices.

– Foreign Exchange rates: a important part of the financial transactions around the world come from the FX investment universe. We retained four key exchange rates for their ample liquidity and well known economic interest: the Euro, the Yen, the Swiss Franc and the British Pound against the Dollar.

– Commodities: we use two additional series of returns from the commodity universe. We focus on the NYMEX crude oil index and on the Dow Jones broad commodity index. The second index is a diversified index representing the whole commodity markets, as it includes both hard and soft commodities.

The dataset starts on April 3, 1998 and ends on December 12, 2010. This period was selelected as it stands a good chance of being stationary: this period is at the end of the disinflationnary period that starts in 1979 with the Volcker era – which is essential in terms of bonds' behavior – and it covers the moment when China joined the World Trade Organization – which is likely to have a tremendous impact in terms of emerging world related assets. The data frequency is weekly: as we focus on regimes, we need to find enough persistence in assets' returns to obtain reliable estimates of the MS parameters. The weekly frequency offers a balanced mix between non Gaussian returns and a stronger persistence than daily data. As in Maheu and McCurdy (2000) or in Maheu et al. (2011) the risk-free rate is not isolated from the equity and commodity returns. On the contrary, in the case of the fixed income universe, we focus on returns in excess of the money market rate.

Descriptive statistics are presented in Table (4). Data are sorted by their Sharpe ratio. From March 1998 to December 2010, the Investment Grade EMU bonds are characterized by the highest return by unit of risk, its Sharpe ratio being equal to 0.261. On the contrary, the exhange rate Swiss Franc againts US Dollar presents the lowest Sharpe ratio, -0.136. Broadly speaking, the bonds, high yield EMU excepted, have a higher Sharpe Ratio – above 0.2 – than the equities, MSCI EM Latam excepted. That means that, in the considered period, it was more interesting to invest in bonds than in equities : the risk premium per unit of risk was the highest. All the considered exchange rates, Euro against US Dollar excepted, are characterized by a negative Sharpe Ratio: the return per unit of risk is negative. In addition to that, all the assets present a negative skewness, reflecting the asymetry of their distribution. The positive excess kurtosis, from 0.26 for the Swiss Franc against Dollar to 25.16 for the high yield US bonds, underlines the thick tails of the assets' distributions.

4 Empirical Results

4.1 Main Results

The main results of our test methodology are presented in Figure (1). This figure provides the estimated number of states for each asset of our sample, following the likelihood ratio methodology, computed at the 1% threshold and for a Newey-West variance estimator with 0 or 10 lags. The main conclusion is that more than two states generally explain asset return dynamics. The outcomes from our statistical tests show that two regimes provide an accurate description of the behavior of returns only in the case of the exchange rates, the commodities and the investment grade US and EMU bonds. On the contrary, every equity indices is characterized by three regimes. Figure (1) also shows that 5 regimes are required to capture the dynamics of the high yield bonds.

In addition to that, the number of regimes does not strongly depend on the way we compute the variance of the likelihood ratio. Consistent with Table (3), not considering lags in the Newey-West variance estimator increases the number of states for a few assets, compared to the Newey-West variance estimator computed with 10 lags.

To check the robustness of this conclusion, we perform a rolling estimation with a rolling window whose starting date is April 03, 1998 and that ends from February 02, 2007 to December 31, 2010. Table (5) provides estimations of the number of regimes for various sub-periods, that is 2007-2008, 2009-2010 and 2007-2010. We find that the number of regimes is stable over the selected periods. Most of the assets have the same number of regimes regardless of the periods. For example, Table (5) shows that, in the case of the likelihood ratio test, with a Newey-West variance estimator performed with 10 lags, Govt. Debt EMU is always driven by two regimes and that Eurostoxx is characterized by a three-regime dynamics. However, moderate differences can be also highlighted. The SP 500 Index is characterized by 2 regimes from 1998 to 2010, however a third state seems to be relevant for the rolling windows 2007-2010 and 2007-2008. Similar results are provided by the likelihood ratio test, with a Newey-West variance estimator performed with 0 lag.

4.2 Does the number of regimes depend on the returns' distributional characteristics?

This subsection tackles the issue of the relation between distributional properties of financial series and the number of regimes: it is possible that additional regimes are necessary to account for strongly non Gaussian returns' distributions. We solve this issue by performing a multivariate probit regression of the number of regimes implicit in asset returns on key statistics describing the distribution of the returns. The probit regression can be described as follows. The number of regimes for asset i, n_i, is related to the random variable $(n^?_i)$ which is unobserved and specified as $n^?_i = \sum_{l=1}^{4} \alpha_l f_{l,i} + \epsilon_i$, where $f_{l,i}$ stands for the centered moment of order l, for the returns of the asset i. To take into account the distributional characteristics of the distribution, the factors $f_{l,i}$ correspond to the volatility, the skewness and the kurtosis of each return. ϵ_i is a Gaussian variable with mean 0 and variance σ^2. The thresholds of the latent variable are characterized as follows:

$$n_i = \begin{cases} 1 & \text{if} \quad n^?_i \leq \nu_1 \\ 2 & \text{if} \quad \nu_1 < n^?_i \leq \nu_2 \\ \vdots & \\ K & \text{if} \quad \nu_{K-1} < n^?_i \end{cases} \tag{9}$$

The unknown parameters α, σ and $(\nu_j)_{j=1,\ldots,K-1}$ are estimated by maximizing the log-likelihood function associated to $(n_i)_i$.

Consistent with this model, Table (6) shows that the number of states does not depend on the distributional characteristics, but in the case of a high number of states, that is 4 to 5 regimes, as the t-value relative to the threshold between 4 or 5 states is equal to 2.9. In addition to that, the table highlights that, when the number of states depends on the distributional characteristics, none of the factors – volatility, skewness or kurtosis – appears to be relevant at the 5% threshold.

4.3 What is the rationale behind a model with more that 2 regimes?

This subsection discusses the economic rationale of a Markov switching model with more than two regimes. Tables (7) and (8) provide the transition probabilities and the distributional characteristics for each asset of our dataset and for each regime. Assets are sorted by their number of regimes, estimated with the likelihood ratio test, performed with a Newey-West variance estimator computed with 0 lag.

The assets driven by a 2-regime model are rather characterized by two very stable regimes. For example, the probability that the DJ Commodity Index remains in regime 1 at time $t+1$, if this regime prevails at time t, p_{11}, is equal to 0.99. Similarly, $p_{22} = 0.93$. Similar results are obtained for the government debt EMU, the investment grade US and the exchange rate Euro against Dollar. On the contrary, for Swiss Franc and Yen against Dollar, one very stable regime prevails. For example, for Yen against Dollar, $p_{11} = 0.99$ whereas $p_{22} = 0.57$. Our conclusion is the following: broadly speaking, these assets are characterized by one main regime. Only major crisis, like the 2008 financial crisis, induce changes in the market conditions for a limited period of time. Figures (2) and (3) illustrate this point. In terms of market characteristics, the stable regimes of Swiss Franc and Yen against Dollar correspond to bull market periods. For Swiss Franc, $\mu_2 = 3\%$ and $\sigma_2 = 0.22$. Similar results are obtained for Yen against US Dollar. For the other assets characterized by a 2-regime model, the two regimes are related to bull and bear markets. For instance, for the DJ Commodity Index, the bull market is characterized by high average returns and low volatility, $\mu_1 = 9\%$ and $\sigma_1 = 0.33$, whereas bear conditions display low returns coupled with high volatility : $\mu_2 = -91\%$ and $\sigma_2 =$ 0.76.

The assets driven by a 3-regime model present one very stable regime and two others that are more volatile. For example, for Eurostoxx or the US Government Bonds, $p_{11} > 0.9$ and $p_{ij} \approx 0.5$, $i = 1$, $j = 1$. This result can be interpreted as follows: the returns are driven by either regime 1 or by the pair regime 2/regime 3 which alternates. Some other assets, like the Investment Grade EMU bonds or the MSCI Europe Index, are characterized by one very stable regime and two other regimes which are quite persistent, the probability of staying in the same regime being close to 0.85. That means that three different regimes really drive the market. Figure (4) illustrates that point. The persistent regime of the assets like Eurostoxx or GBP/USD exchange rate is rather a bear regime, characterized by a negative average return. On the contrary, the persistent regime of the assets like the Government Debt US, MSCI EM Asia or MSCI EM Europe corresponds to a bull market. As expected, the other regimes are related to bull and bear markets, which are more or less transitory. For example, the most persistent regime of the MSCI EM Asia is characterized by an annualized average return equal to +41% – and an annualized volatility equal to 0.29 – whereas the annualized characteristics of the other regimes are respectively (174%,0.53) and (-48%,0.59).

The assets characterized by more than three regimes always present two persistent regimes. For example, for SP500, $p_{22} = 0.84$ and $p_{33} = 0.99$. The other regimes often are transitory states. For the MSCI EM Latam Index, $p_{14} = 0.96$, $p_{41} = 0.49$ and $p_{44} = 0.32$. That means that if the market is in regime 1 at time t, it will switch to regime 4 at time $t+1$ with a probability of 0.96. Being in this regime, it will alternate between regime 4 and regime 1. In the case of SP500, similar results are reached for regimes 1 and 4. The market is driven either by regime 3 or by jumps from regime 1 to regime 4. Regime 2 corresponds to pure crisis conditions. Regime 3 is a bull regime, with yearly returns equal to +12%. Regimes 1 and 4 correspond to range trading states: $\mu_1 = -59\%$ and $\mu_4 = +96\%$. On the contrary, regime 2 is characterized by the lowest returns and the highest volatility: $\mu_2 = -81\%$ and $\sigma_2 = 1.04$. Figure (5) illustrates these elements.

5 Mesuring the economic value of a higher number of regimes

5.1 Predicting the risk premium

After having identified the number of regimes driving the asset returns, we are now interested in the performance of a strategy which would take into account this information, compared to a strategy based on a 2-regime model, so that to gauge the economic interest of our previous detailed examination.

We compare the return of two portfolio strategies based either on a model with 2 regimes or a model with $\hat{n}$ regimes, $\hat{n}$ being the estimated number of regimes. More precisely, at each date t of the out-of-sample period, we estimate the Markov switching parameters of the underlying assets. Knowing the transition probabilities, we can then forecast the probabilities associated with each state for next week – by using Equation (4) – and thus an expected return. When this expected return is positive (resp. negative), we go long (resp. short). Each strategy's volatility is equal to 10%. It would be possible to consider other rules. For example, it would be possible to build the strategy on the highest probability : if the expected return associated with its corresponding state is positive (resp. negative), we go long (resp. short). Nevertheless, such a strategy would be difficult to justifiy in the case of assets having a high number of regimes. If an asset is characterized by more than two regimes, it is possible to obtain probabilities of less than 50% for all the regimes. It is thus difficult to assess their contribution to the returns' distribution. Using the expected return allows integrating all the available information, as the expected return is a combination of each probability.

Without considering trading costs, Table (9) highlights that a multi-regime based strategy allows improving the performance compared to a two-regime based strategy. More precisely, from 2007 to 2010, taking into account the right number of regimes allows reaching a better or an equivalent return when compared to a two-regime based strategy for 15 assets of our sample. Only 4 assets are penalized. In addition to that, the strategy yields a positive return for the high yield bonds – US and EMU – and for the MSCI Small Caps EMU Index whereas it is negative when considering only 2 regimes. Moreover, the assets characterized by the highest number of regimes are also characterized by the best performances (+30% and +35% for the high yield EMU and US bonds) when considering the estimated number of regimes.

5.2 Distributional issues

Beyond forecasting expected returns, we now evaluate density forecasts based either on the estimated number of regimes of the underlying return distribution or on the two regimes case. The underlying goal is to evaluate if taking into account the estimated number of regimes improves the forecasting performance compared to a two regimes model.

Our forecasting experience is based on a rolling window. First, we estimate the number of regimes, the parameters of the MS model based on this estimated number of regimes and the parameters of a MS model with two regimes. The in-sample period used to perform such estimations always starts on March 1998, as in the previous empirical applications. The size of this in-sample period is increasing : the first window ends on January 1st of 2007 and the last ones incorporates most of the total sample's observations. Given these rolling estimations, we forecast the density of returns from one to four weeks ahead. More formally, we compare the density forecasting performances of both models by using a Diebold and Mariano (1995) metric.

We compute the average predictive likelihood over the out-of -sample observations $t = \tau + k_{max}, \ldots, T$, that is:

$$L_{M,k} = \frac{1}{T - \tau - k_{max} + 1} \sum_{t=\tau+k_{max}-k}^{T-k} \log f_{M,k}\left(r_{t+k}|\widehat{\theta}_M\right), \tag{10}$$

where k is the forecast horizon, τ is the number of observations used for the first in-sample window, T is the total number of observations, $f_{M,k}$ is the forecast density k weeks ahead for model M using the estimated parameters $\widehat{\theta}_M$ and k_{max} is equal to four weeks. We compared the forecasting power of model $MS(\hat{n})$ and model $MS(2)$ through the following test statistics:

$$t^k_{MS(\hat{n}),MS(2)} = \frac{L_{MS(\hat{n}),k} - L_{MS(2),k}}{\frac{\hat{\sigma}_{MS(\hat{n}),MS(2),k}}{\sqrt{T-\tau-k+1}}}. \tag{11}$$

It is assumed to be asymptotically standard Normal. Following Diebold and Mariano (1995)'s test, a significant positive (negative) estimated value rejects the null of equal performance between competing forecasts, and provides evidence in favor of model $MS(\hat{n})$ ($MS(2)$).

Table (10) provides the p-values of the tests. All in all, for most cases, considering the right number of regimes either improves or delivers a similar performance in terms of density forecast when compared to the two regime case. First of all, the results appear to be independant of the forecasting horizon. For instance, the SP500 p-value computed with LR(0) is in a range of 2.916 to 2.923 from H+1 to H+4. Then, without considering the horizon issue, we can highlight that the density forecast performance depends on the way we compute the Newey-West variance estimator. When considering 10 lags, and for a given horizon, the density forecast of the model based on the estimated number of regimes is better for four cases than the density forecast of the model based on two regimes. This is identical for seven cases. Lastly, the density forecast performance of the two-regimes model is better for three assets. When considering 0 or 10 lags, the density forecast performance of the 2-regimes model is better for the same assets. Nevertheless, for 8 assets, the Diebold and Mariano (1995)'s test does not allow discriminating between the two models. With LR(0) the estimation of the right number of regimes improves the density forecast for two assets and makes it worse for two assets. In the case of assets characterized by the highest number of regimes, that is high yield US and EMU bonds, the density forecast performances are better with the two-regime model than with the model based on the estimated number of regimes. This result should be related to the underlying parameter estimation difficulties: the higher the number of regimes, the higher the number of parameters to be estimated, and the higher the estimated parameters' volatility.

6 Conclusion

In this paper, we show that contrary to a shared belief, more than two states generally explain asset returns' dynamics. This assumption can only be considered as valid for a few assets: exchange rates, commodities and US investment grade EMU bonds. On the contrary, equity indices are rather characterized by three regimes and five regimes are required to capture the dynamics of the high yield bonds. But for a high number of regimes, the number of regimes only weakly depends on the statistical properties of the underlying asset.

Based on this result, we compare out-of-sample risk premium forecasts and base an active investment strategy on that: our results confirm the over performance of the forecasts obtained when estimating the right number fo regimes rather than when setting it ex ante to be equal to two. Lastly, the density forecasts obtained with the estimated number of regimes are equivalent or better than the ones obtained with a two-regime model.

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A Tables and Figures

	Expected Return	Volatility
Regime 1	-0.92%	5.98%
Regime 2	0.23%	2.61%
Regime 3	0.12%	2.19%
Regime 4	0.29%	1.30%

Table 1: MC distribution characteristics.

	State 1	State 2	State 3	State 4
State 1	86.18%	6.91%	0.00%	6.91%
State 2	1.40%	96.80%	0.00%	1.80%
State 3	0.90%	0.00%	94.30%	4.80%
State 4	0.00%	0.03%	3.70%	96.27%

Table 2: MC transition probabilities.

	% correctly estimated number of regimes	% underestimated number of regimes	% overestimated number of regimes
PS (2003) Test 1	9.5%	89.2%	1.3%
PS (2003) Test 2	1.4%	98.6%	0.0%
PS (2003) Test 3	3.6%	96.3%	0.1%
Smith et al. 2005	7.2%	45.2%	47.5%
LR-NW test 1%	72.8%	26.0%	1.2%
LR-NW test 5%	72.8%	15.4%	11.7%
LR-VW test 10%	58.4%	11.4%	30.2%
LR-Other test 1%	80.0%	18.9%	1.1%
LR-Other test 5%	74.8%	13.3%	11.9%
LR-Other test 10%	57.9%	9.7%	32.3%

Table 3: Monte-Carlo investigation of the different test methodologies.

Assets	Average return (ann.)	Volatility (ann.)	Sharpe Ratio	Skewness	Kurtosis
Investment Grade EMU	1.90%	7.27%	0.261	-1.06	5.69
Investment Grade US	2.93%	12.38%	0.237	-0.89	5.41
Govt. Debt US	2.41%	10.70%	0.225	-0.46	1.06
MSCI EM Latam	12.26%	55.78%	0.220	-0.51	4.16
Govt. Debt EMU	1.80%	8.85%	0.204	-0.54	2.38
High Yield US	3.32%	18.05%	0.184	-2.31	25.16
MSCI EM Europe	9.62%	70.48%	0.137	-0.31	7.90
WTI	7.94%	59.41%	0.134	-0.70	2.83
MSCI EM Asia	5.96%	51.25%	0.116	-0.45	2.03
EURUSD	1.45%	22.56%	0.064	-0.23	0.90
MSCI Small Cap EMU	2.68%	48.99%	0.055	-1.47	7.41
DJ Commodity	1.93%	38.90%	0.050	-0.87	3.28
Russell 2000	2.35%	55.08%	0.043	-0.64	3.56
High Yield EMU	0.97%	25.65%	0.038	-1.41	12.83
SP500	-0.04%	44.13%	-0.001	-0.76	5.93
Eurostoxx	-1.39%	53.46%	-0.026	-0.77	5.83
GBPUSD	-0.63%	21.64%	-0.029	-0.61	4.21
USDJPY	-3.40%	27.21%	-0.125	-1.39	10.32
USDCHF	-3.21%	23.54%	-0.136	-0.19	0.26

Table 4: Descriptive statistics of the returns on the assets considered in the dataset

The statistics presented in the table are computed using logarithmic returns over the period that starts on 04/03/1998 and ends on 12/31/2010. The data frequency is weekly. Both the standard deviation and the average returns are scaled into yearly quantities for ease of reading.

	Empirical LR test 1% (Lag NW = 10)				Empirical LR test 1% (Lag NW = 0)			
	Total Sample Estimation	Median over 2007-2010	Median over 2007-2008	Median over 2009-2010	Total Sample Estimation	Median over 2007-2010	Median over 2007-2008	Median over 2009-2010
SP500	2	3	3	2	4	3	3	3
Investment Grade US	2	2	3	2	2	2	3	2
Govt. Debt EMU	2	2	2	2	2	2	2	2
WTI	2	2	2	2	3	2	2	3
DJ Commodity	2	2	2	2	2	2	2	3
MSCI EM Latam	2	2	2	2	4	3	3	3
MSCI EM Europe	2	2	2	2	3	3	3	3
USDCHF	2	2	2	2	2	2	2	2
EURUSD	2	2	2	2	2	2	2	2
USDJPY	2	2	2	2	2	2	2	2
GBPUSD	2	2	2	2	3	2	2	3
Govt. Debt US	3	2	2	3	3	2	2	3
Russell 2000	3	2	2	2	3	2	2	3
Eurostoxx	3	3	3	3	3	3	3	3
MSCI Small Cap EMU	3	3	2	3	3	3	3	3
Investment Grade EMU	3	2	2	3	3	2	2	3
MSCI EM Asia	3	3	3	3	3	3	3	3
High Yield US	5	5	5	5	5	5	5	6
High Yield EMU	5	5	4	5	5	5	5	5

Table 5: Estimated number of regimes, following Empirical LR at 1% level (Lag NW = 10 and Lag NW = 0)

Parameter Estimates			
	Value	Std. Error	t-value
Volatility	3.1	2.6	1.2
Skewness	0.6	2.0	0.3
Kurtosis	0.4	0.2	1.6
Threshold Estimates			
2—3	1.5	1.5	1.0
3—4	4.4	1.8	2.5
4—5	5.9	2.0	2.9

Table 6: Multivariate probit estimates of the factors explaining the number of regimes in asset returns

This table presents the results of a multivariate probit regression trying to explain the number of regimes implicit in asset returns. The top panel presents the estimates and their standard deviations whereas the bottom panel presents the estimated thresholds.

	DJ Commodity	EURUSD	Govt. Debt EMU	Investment Grade US	USDCHF	USDJPY	Eurostoxx	GBPUSD	Govt. Debt US	Investment Grade EMU	MSCI EM Asia	MSCI EM Europe	MSCI Small Cap EMU	Russell 2000	WTI	MSCI EM Latam	SP500	High Yield EMU	High Yield US
p_{11}	0.99	0.99	0.93	0.94	0.45	0.99	0.93	0.32	0.98	0.97	0.88	0.87	0.86	0.23	0.67	0.02	0.51	0.86	0.94
p_{12}	0.01	0.01	0.07	0.06	0.55	0.01	0.07	0.68	0.02	0.02	0.12	0.12	0.13	0.76	0.31	0.01	0.03	0	0
p_{13}							0	0	0	0.01	0	0.01	0.01	0	0.02	0.01	0	0.05	0
p_{14}																0.96	0.46	0.09	0
p_{15}																		0	0.06
p_{21}	0.07	0.03	0.02	0.01	0.04	0.43	0	0.27	0.02	0.02	0.46	0.07	0.18	0.38	0.26	0.07	0	0	0
p_{22}	0.93	0.97	0.98	0.99	0.96	0.57	0.45	0.72	0.46	0.83	0.04	0.93	0.77	0.6	0.74	0.93	0.84	0.91	0.81
p_{23}							0.55	0	0.52	0.15	0.49	0	0.05	0.02	0	0	0	0.06	0.03
p_{24}																0	0.16	0	0.06
p_{25}																		0.02	0.09
p_{31}							0.04	0.04	0	0.01	0.07	0.15	0.1	0.07	0.04	0.13	0.01	0.06	0
p_{32}							0.59	0	0.48	0.13	0	0	0	0	0.01	0	0	0.04	0
p_{33}							0.37	0.96	0.52	0.86	0.93	0.85	0.9	0.93	0.95	0.87	0.99	0.8	0.95
p_{34}																0	0	0	0
p_{35}																		0.1	0.05
p_{41}																0.49	0.73	0	0
p_{42}																0	0	0	0.26
p_{43}																0.19	0.01	0.05	0.09
p_{44}																0.32	0.26	0.9	0.65
p_{45}																		0.05	0
p_{51}																		0	0.01
p_{52}																		0.08	0.02
p_{53}																		0.1	0.01
p_{54}																		0.01	0.16
p_{55}																		0.81	0.8

Table 7: Transition probabilities.

This table providesthe probability to switch from the state i to the state j for each asset. For instance, in the case of the DJ commodity, $p_{11} = 0.99$ and $p_{12} = 0.01$ means that, if at time t the DJ commodity return is driven by regime 1, it will be driven by the same regime at time $t + 1$ with a probability of 0.99 and by regime 2, with a probability of 0.01.

	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5
DJ Commodity	0.09	0.33	-0.91	0.76						
EURUSD	0.05	0.19	-0.1	0.3						
Govt. Debt EMU	-0.01	0.13	0.03	0.07						
Investment Grade US	0.02	0.22	0.03	0.1						
USDCHF	-0.82	0.28	0.03	0.22						
USDJPY	-0.01	0.23	-0.92	0.86						
Eurostoxx	-0.32	0.9	0.8	0.26	-0.76	0.32				
GBPUSD	0.39	0.14	-0.14	0.17	-0.21	0.48				
Govt. Debt US	0.01	0.07	0.2	0.08	-0.12	0.14				
Investment Grade EMU	-0.01	0.11	0.1	0.04	-0.03	0.06				
MSCI EM Asia	-0.48	0.59	1.74	0.53	0.41	0.29				
MSCI EM Europe	-0.14	0.8	0.37	0.41	-2.64	2.16				
MSCI Small Cap EMU	0.53	0.24	-0.56	0.31	-0.38	0.88				
Russell 2000	0.9	0.31	-0.31	0.41	-0.46	1.03				
WTI	0.88	0.32	-0.36	0.46	-0.36	0.97				
MSCI EM Latam	-0.49	0.38	-0.67	1.32	0.59	0.3	0.02	0.64		
SP500	-0.59	0.34	-0.81	1.04	0.12	0.23	0.96	0.34		
High Yield EMU	-0.92	0.48	0.09	0.04	-0.29	0.12	0.74	0.32	0.35	0.12
High Yield US	-0.13	0.56	0.22	0.06	0	0.05	0.46	0.13	-0.33	0.13

Table 8: Asset characteristics for each asset and each regime.

This table provides the market characteristics for each asset and each regime from 1998 to 2010. μ_j and σ_j stand for the mean and volatility of the underlying asset for regime j. For instance, μ_1 and σ_1 of the SP500 are equal to -0.59 and 0.34: that means that the yearly average return in regime 1, of the SP500 is equal to -59% and that the yearly volatility, for the same regime, is equal to 34%.

Asset	LR test 1% (Lag = 10)	Buy and Hold	2 Regimes
Eurostoxx	-5.88%	-1.08%	-2.12%
Investment Grade EMU	-4.09%	1.46%	-3.39%
Govt. Debt US	-0.99%	7.40%	-0.99%
EURUSD	-0.76%	0.22%	-0.76%
Govt. Debt EMU	-0.47%	1.41%	-0.47%
GBPUSD	0.92%	-4.41%	0.92%
DJ Commodity	2.04%	-6.84%	2.04%
MSCI EM Europe	2.32%	-2.71%	2.32%
MSCI EM Latam	2.70%	3.86%	2.70%
Investment Grade US	3.90%	5.26%	3.90%
Russell 2000	3.99%	-9.36%	5.54%
USDCHF	4.36%	4.36%	4.36%
MSCI EM Asia	5.79%	2.76%	6.16%
WTI	6.14%	-0.73%	6.14%
USDJPY	6.74%	6.74%	6.74%
MSCI Small Cap EMU	7.49%	-8.86%	-0.11%
SP500	10.42%	-5.71%	8.51%
High Yield EMU	30.29%	2.01%	-3.60%
High Yield US	35.75%	2.90%	-1.40%

Table 9: The economic value of a multi-regime based strategy.

	Horizon = +1		Horizon = +2		Horizon = +3		Horizon = +4	
	LR(0)	LR(10)	LR(0)	LR(10)	LR(0)	LR(10)	LR(0)	LR(10)
SP500	2.916	4.019	2.920	4.030	2.922	4.038	2.923	4.044
Investment Grade US	-1.078	-1.078	-1.113	-1.113	-1.140	-1.140	-1.168	-1.168
Govt. Debt EMU	–	–	–	–	–	–	–	–
WTI	1.078	2.896	1.026	2.830	0.963	2.779	0.903	2.740
DJ Commodity	–	1.466	–	1.582	–	1.670	–	1.736
MSCI EM Latam	0.971	3.937	0.971	3.975	0.957	3.927	0.956	3.947
MSCI EM Europe	-1.027	0.561	-1.027	0.556	-1.027	0.554	-1.027	0.554
USDCHF	–	–	–	–	–	–	–	–
EURUSD	–	–	–	–	–	–	–	–
USDJPY	–	–	–	–	–	–	–	–
GBPUSD	–	1.376	–	1.377	–	1.378	–	1.379
Govt. Debt US	-2.403*	-2.403*	-2.406*	-2.406*	-2.405*	-2.405*	-2.402*	-2.402*
Russell 2000	1.596	2.402	1.591	2.388	1.586	2.375	1.582	2.364
Eurostoxx	1.318	1.239	1.320	1.241	1.322	1.243	1.319	1.241
MSCI Small Cap EMU	2.454	3.556	2.439	3.489	2.422	3.434	2.405	3.390
Investment Grade EMU	-0.164	-0.593	-0.108	-0.548	-0.062	-0.511	-0.020	-0.477
MSCI EM Asia	-0.639	-0.639	-0.630	-0.630	-0.623	-0.623	-0.616	-0.616
High Yield US	-3.762*	-4.796*	-3.443*	-4.573*	-3.228*	-4.389*	-3.072*	-4.232*
High Yield EMU	-3.136*	-3.172*	-3.054*	-3.099*	-2.981*	-3.035*	-2.915*	-2.977*

Table 10: Likelihood ratio density forecast tests.

The results presented in this table correspond to the p-values of the Diebold and Mariano (1995) test. The number of regimes is estimated with the likelihood ratio tests performed with a Newey-West variance estimator computed with 0 lag (LR(0)) or 10 lags (LR(10)). Bold fonts stand for the case where the p-values are higher than 1.96: the performance of the density forecast based on the estimated number of regimes is better than the density forecast based on two regimes. "*" stand for the case where the p-values are lower than -1.96: the performance of the density forecast based on two regimes is better than the performance based on the estimated number of regimes.
"–" stands for the case where two regimes are already estimated.

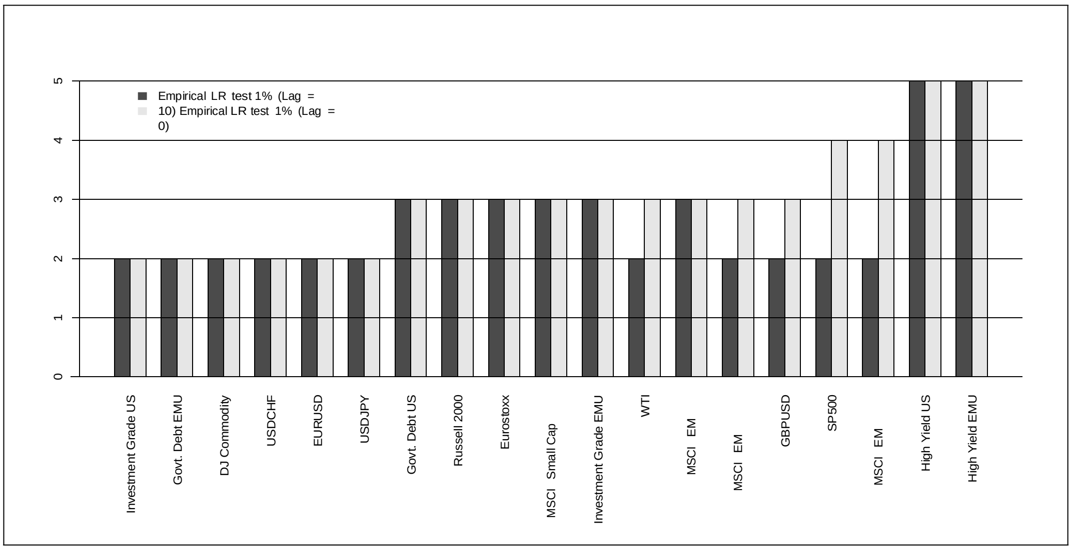

Figure 1: Estimated number of regimes in asset returns

This figure presents the estimated number of regimes for each asset. Lag $= 0$ or Lag $= 10$ indicates the number of lags included in Newey-West variance estimator.

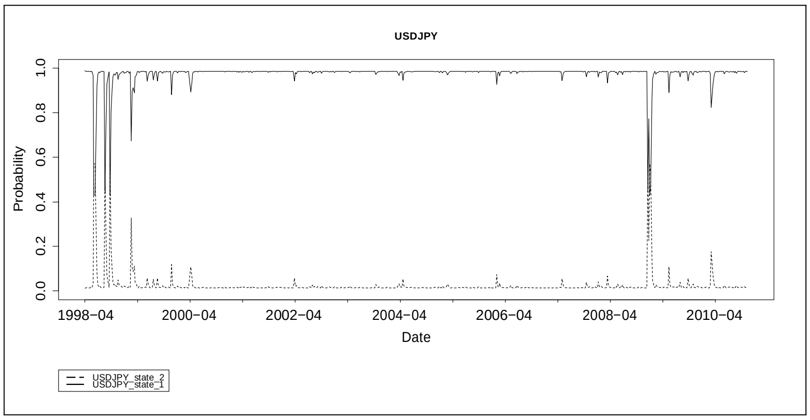

Figure 2: State probability for USD/Yen exchange rate from 1998 to 2010

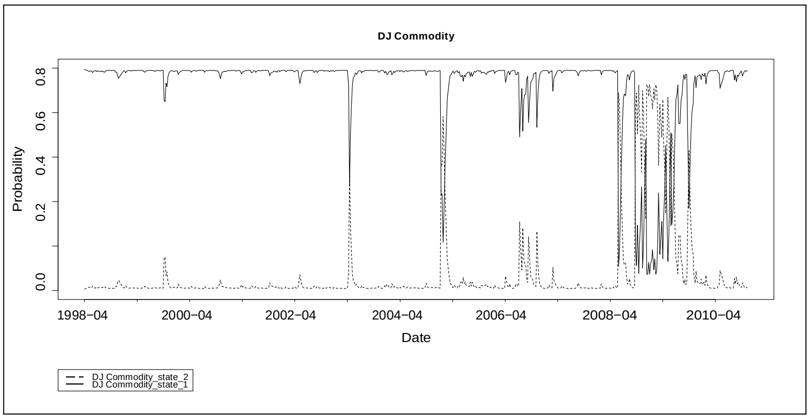

Figure 3: State probability for DJ Commodity from 1998 to 2010

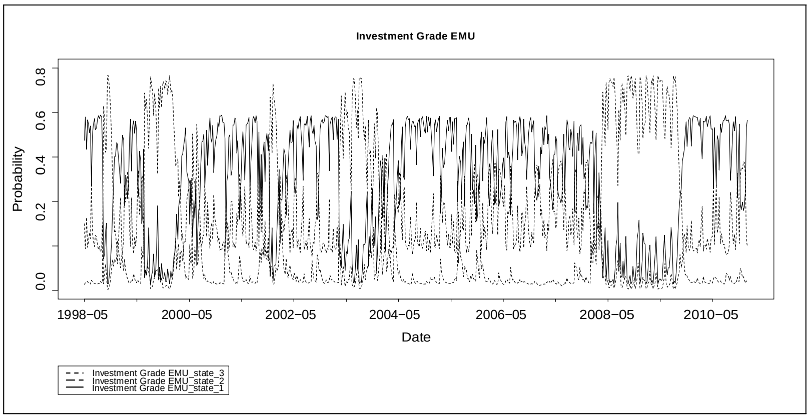

Figure 4: State probability for the Investment Grade EMU Bonds from 1998 to 2010

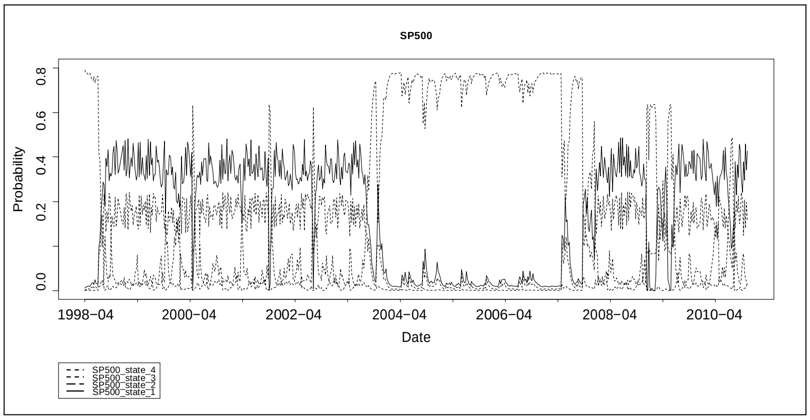

Figure 5: State probability for the SP500 Index from 1998 to 2010